B.Sc Sem IV Date ....

Expt. No. .... Analysis II – (C-8) Page No. ....

DR. Anita Kumari

Relation between Beta & Gamma Functions

Prove that
$$B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$

We have
$$\frac{\Gamma(m)}{z^m} = \int_0^\infty x^{m-1} e^{-zx}\, dx$$

$$\therefore \quad \Gamma(m) = \int_0^\infty z^m x^{m-1} e^{-zx}\, dx$$

Multiplying both sides by $e^{-z} z^{n-1}$, we get
$$\Gamma(m)\, e^{-z} z^{n-1} = \int_0^\infty z^{m+n-1} e^{-z(1+x)} x^{m-1}\, dx$$

Now integrating both sides w.r.t z from 0 to ∞, we have
$$\Gamma(m) \int_0^\infty e^{-z} z^{n-1}\, dz = \int_0^\infty \left[\int_0^\infty z^{m+n-1} \cdot e^{-z(1+x)} \right] x^{m-1}\, dx$$

$$\Rightarrow \Gamma(m)\,\Gamma(n) = \int_0^\infty \frac{\Gamma(m+n)}{(1+x)^{m+n}} \cdot x^{m-1}\, dx$$

$$= \Gamma(m+n) \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx$$

$$\Rightarrow \Gamma(m)\,\Gamma(n) = \Gamma(m,n)\; B(m,n)$$

Teacher's Signature : ....

$$\therefore \boxed{B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}}$$

Hence proved.